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| BIE-PKM Final Test, FIT CTU, ZS 2023/2024 | Date: |
| Name and Surname: | Points: |

You have 70 minutes to complete the test. Calculators or other devices are not allowed.

Question 1 (2p). Consider sets $A = \{2, 4, 8\}$ and $B = \{2, 4, 8, 16, 32\}$. Which of the statements below are true of A and B ?

1. All numbers in A are also in B .
2. All numbers in A and in B are even.
3. Some numbers in B are also in A .
4. Some number in B is not in A .

Question 2 (2p). Choose correct statements about a_k , $k = 1, 2, \dots$, the terms of a geometric sequence with common ratio q .

1. $a_k = a_{k-1}q$, $k = 2, 3, \dots$
2. $a_k = a_{k-1}q$, $k = 1, 2, 3, \dots$
3. $a_k = a_1q^k$, $k = 1, 2, 3, \dots$
4. $a_k = a_1q^{k-1}$, $k = 1, 2, 3, \dots$

Question 3 (2p). Find all real solutions of the inequation

$$a^x < \frac{1}{a^2},$$

where a is a real number from the interval $(0, 1)$.

1. $x \in (2, +\infty)$
2. $x \in (-\infty, -2)$
3. $x \in (-2, +\infty)$
4. $x \in (-\infty, 2)$

Question 4 (2p). There are 50 guests at a wedding banquet. Each guest will shake hands with all other guests exactly once. How many handshakes occur in the course of this celebration?

1. 1225
2. 2450
3. 2500
4. 1176

Question 5 (2p). We consider three arbitrary lines in a plane. Which of the following situations may never occur?

1. The union of all points of intersection of these lines contains exactly one points.

2. The union of all points of intersection of these lines contains exactly four points.
3. The union of all points of intersection of these lines contains exactly two points.
4. The union of all points of intersection of these lines contains exactly three points.

Question 6 (2p). Say whether the function

$$f(x) = x^4 + 4x^2 + 3$$

is even or odd.

Question 7 (2p). Determine all real solutions of the equation

$$x^3 - 4x^2 + x + 6 = 0,$$

if you know that 2 is a root of the polynomial on the left.

Question 8 (2p). Is this argument correct? "No arts students can code. Maria can code. So, Maria is not an arts student."

Question 9 (2p). Prove the formula

$$\frac{2\operatorname{tg}(x)}{1 + \operatorname{tg}^2(x)} = \sin(2x).$$

For which $x \in \mathbb{R}$ the above formula applies (i.e. it is well defined)?

Question 10 (2p). We have two lines given by equations $3x - 2y = 7$ and $4x + 3y = 15$. Decide if they are parallel or if they intersect, and what is their intersection.
