Personalized Machine Learning

Autoencoders for CF

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October 8, 2024

Dimensionality Reduction

- Dimensionality reduction is a technique used to reduce the number of features in a dataset while retaining the most relevant information.
- Different types of data inputs, such as music, photos, or text, have unique characteristics that require specific machine learning approaches.
- Traditional methods like PCA may fail, particularly when dealing with data featuring non-linear relationships.
- Autoencoders, which are unsupervised neural networks, are employed for compressed data representation and are effective for dimensionality reduction and handling complex inputs.

Autoencoder

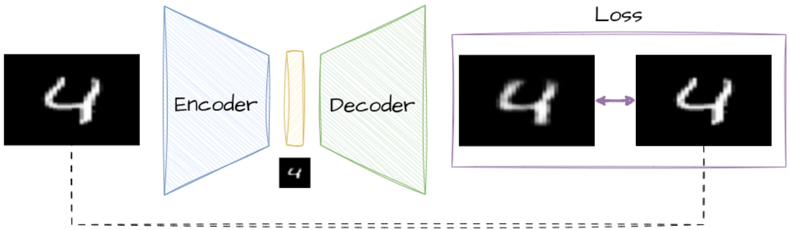
- An autoencoder is a type of feed-forward neural network.
- It is designed to reconstruct its input x_i as output x_i.
- Traditional methods like PCA may struggle, especially when dealing with non-linear relationships.
- To prevent trivial solutions, the network includes a bottleneck layer (or code layer) with significantly fewer dimensions than the input.

Autoencoder

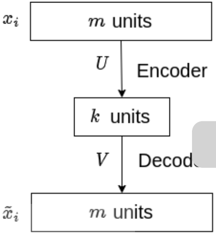
- An autoencoder is composed of both an encoder and a decoder.
- The encoder and the decoder typically have a similar structure.
- More formally, let $\mathcal{E}(x)$ be an encoder and $\mathcal{D}(x)$ be a decoder. Our optimization problem can be described as follows:

$$\min_{\mathcal{E},\mathcal{D}} \sum_i ||x_i - \mathcal{D}(\mathcal{E}(x_i))||$$

What is an autoencoder?



Autoencoders



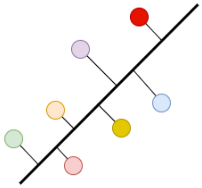
- The simplest autoencoder has one hidden layer and uses squared error loss.
- The optimization function can be denoted as:

$$\min_{U,V} \sum_i ||x_i - x_i UV||^2$$

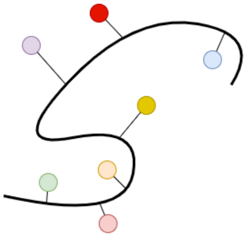
What happens if $k \geq m$?

- If $k < m$, then $UV \neq I$, where I represents the identity function.
- Therefore, if $k \geq m$, any simple solution where $UV = I$ is a trivial solution.
- More importantly, in the trivial case, there is no reduction of dimension.

PCA × Autoencoder

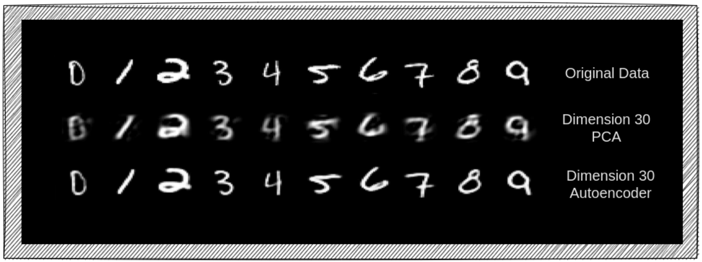


PCA



Autoencoder

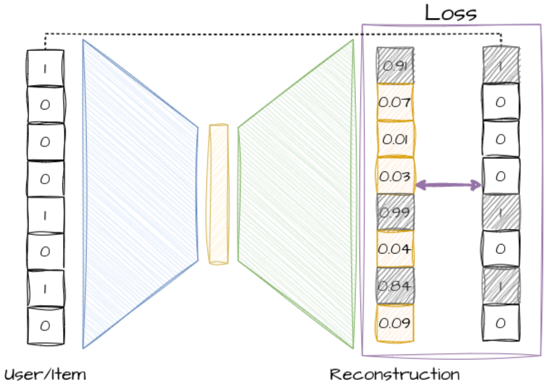
PCA × Autoencoder



Autoencoders for Collaborative Filtering

- Encode user-item interactions into a lower-dimensional space to discover meaningful patterns.
- Applicable to both explicit and implicit feedback.
- Effectively handle sparse user-item interaction data.
- Balancing model complexity and scalability, especially in large-scale recommendation systems.
- Potential for leveraging transfer learning to enhance content-based methods.

Autoencoder for implicit feedback



EASE

- EASE is the shallowest auto-encoder possible.
- It aims to solve the following problem:

$$\text{min}_B ||X - XB||^2 + \lambda ||B||^2 \text{ s.t. } \text{diag}(B) = 0$$

- Why do we need the constraint $\text{diag}(B) = 0$?
- EASE has a closed-form solution, which we will explore shortly.
- Is this a good method?

EASE Results

Table 1: Ranking accuracy (with standard errors of about 0.002, 0.001, and 0.001 on the *ML-20M*, *Netflix*, and *MSD* data, respectively), following the experimental set-up in [13].

(a) *ML-20M*	Recall@20	Recall@50	NDCG@100
popularity	0.162	0.235	0.191
EASE^R	0.391	0.521	0.420
$\text{EASE}^R \geq 0$	0.373	0.499	0.402
results reproduced from [13]:			
SLIM	0.370	0.495	0.401
WMF	0.360	0.498	0.386
CDAE	0.391	0.523	0.418
MULT-VAE^{PR}	0.395	0.537	0.426
MULT-DAE	0.387	0.524	0.419
(b) *Netflix*			
popularity	0.116	0.175	0.159
EASE^R	0.362	0.445	0.393
$\text{EASE}^R \geq 0$	0.345	0.424	0.373
results reproduced from [13]:			
SLIM	0.347	0.428	0.379
WMF	0.316	0.404	0.351
CDAE	0.343	0.428	0.376
MULT-VAE^{PR}	0.351	0.444	0.386
MULT-DAE	0.344	0.438	0.380
(c) *MSD*			
popularity	0.043	0.068	0.058
EASE^R	0.333	0.428	0.389
$\text{EASE}^R \geq 0$	0.324	0.418	0.379
results reproduced from [13]:			
SLIM	— did not finish in [13] —		
WMF	0.211	0.312	0.257
CDAE	0.188	0.283	0.237
MULT-VAE^{PR}	0.266	0.364	0.316
MULT-DAE	0.266	0.363	0.313

EASE: dimensions

$$
\left|\left| \; X - X \times B \; \right|\right|_{\mathrm{Fro}}^{2}
$$

$$
\begin{array}{c|ccc|}
 & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \\
\hline
\mathcal{U}_1 & 1 & 1 & 0 \\
\mathcal{U}_2 & 1 & 1 & 0 \\
\mathcal{U}_3 & 0 & 1 & 1 \\
\mathcal{U}_4 & 0 & 1 & 1 \\
\mathcal{U}_5 & 0 & 0 & 1 \\
\mathcal{U}_6 & 1 & 0 & 1 \\
\hline
\end{array}
\quad - \quad
\begin{array}{c|ccc|}
 & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \\
\hline
\mathcal{U}_1 & 1 & 1 & 0 \\
\mathcal{U}_2 & 1 & 1 & 0 \\
\mathcal{U}_3 & 0 & 1 & 1 \\
\mathcal{U}_4 & 0 & 1 & 1 \\
\mathcal{U}_5 & 0 & 0 & 1 \\
\mathcal{U}_6 & 1 & 0 & 1 \\
\hline
\end{array}
\quad \times \quad
\begin{array}{c|ccc|}
\hline
\mathcal{I}_1 & 0 & & \\
\mathcal{I}_2 & & 0 & \\
\mathcal{I}_3 & & & 0 \\
\hline
 & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3
\end{array}
$$

$$
\mathrm{diag}(B) = 0
$$

EASE: intuition

$$X = \begin{array}{c|ccc} & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \\ \hline \mathcal{U}_1 & 1 & 1 & 0 \\ \mathcal{U}_2 & 1 & 1 & 0 \\ \mathcal{U}_3 & 0 & 1 & 1 \\ \mathcal{U}_4 & 0 & 1 & 1 \\ \mathcal{U}_5 & 0 & 0 & 1 \\ \mathcal{U}_6 & 1 & 0 & 1 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{array}{c|ccc} \mathcal{I}_1 & 0 & 0.66 & 0.33 \\ \mathcal{I}_2 & 0.5 & 0 & 0.5 \\ \mathcal{I}_3 & 0.25 & 0.50 & 0 \\ \hline & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \end{array}$$

EASE: closed-form

$$\min_B \|X - XB\|_F^2 + \lambda \|B\|_F^2$$
$$\text{s.t. diag(B)} = 0$$

Here $X \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times n}$.

Lagrangian:

$$\mathcal{L}(B) = \|X - XB\|_F^2 + \lambda \|B\|_F^2 + 2\gamma^\top \text{diag}(B)$$

EASE: closed-form

Lagrangian:

$$\mathcal{L}(B) = \|X - XB\|_F^2 + \lambda\|B\|_F^2 + 2\gamma^\top \text{diag}(B)$$

Derivative of the Lagrangian:

$$\begin{aligned}
\mathcal{L}'(B) \quad &= 2(X)^\top(XB - X) + 2\lambda B + 2\text{diagMat}(\gamma) \\
&= 2X^\top(XB - X) + 2\lambda B + 2\text{diagMat}(\gamma) \\
&= 2X^\top XB - 2X^\top X + 2\lambda B + 2\text{diagMat}(\gamma)
\end{aligned}$$

EASE: closed-form

Derivative of the Lagrangian:

$$\begin{aligned}
0 &= 2X^\top XB - 2X^\top X + 2\lambda B + 2\text{diagMat}(\gamma) \\
0 &= X^\top XB - X^\top X + \lambda B + \text{diagMat}(\gamma) \\
X^\top XB + \lambda B &= X^\top X - \text{diagMat}(\gamma) \\
(X^\top X + \lambda I)B &= X^\top X - \text{diagMat}(\gamma) \\
\hat{B} &= (X^\top X + \lambda I)^{-1}(X^\top X - \text{diagMat}(\gamma))
\end{aligned}$$

EASE: closed-form

Assume:

$$P = (X^\top X + \lambda I)^{-1}$$

We have:

$$\begin{aligned}
\hat{B} &= (X^\top X + \lambda I)^{-1}\left(X^\top X - \text{diagMat}(\gamma)\right) \\
&= P\left(P^{-1} - \lambda I - \text{diagMat}(\gamma)\right) \\
&= I - P\left(\lambda I + \text{diagMat}(\gamma)\right)
\end{aligned}$$

EASE: closed-form </>

We know that:

$$\text{diag}(\hat{B}) = 0$$

Therefore:

$$\begin{aligned}
\text{diag}\Big(I - P(\lambda I + \text{diagMat}(\gamma))\Big) &= \vec{0} \\
\text{diag}(I) - \text{diag}\big(P \times \text{diagMat}(\lambda\vec{1} + \gamma)\big) &= \vec{0}
\end{aligned}$$

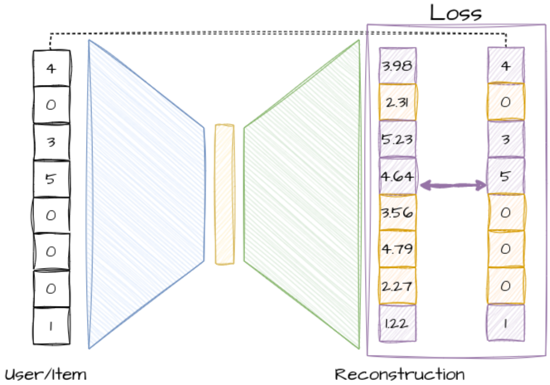
Finally:

$$\begin{aligned}
\vec{1} - \text{diag}(P) \bigodot (\lambda\vec{1} + \gamma) &= \vec{0} \\
\vec{1} - \text{diag}(P) \bigodot \lambda\vec{1} - \text{diag}(P) \bigodot \gamma &= \vec{0} \\
\text{diag}(P) \bigodot \gamma &= \vec{1} - \lambda\text{diag}(P) \\
\gamma &= (\vec{1} - \lambda\text{diag}(P)) \oslash \text{diag}(P)
\end{aligned}$$

Matrix $\hat{B}$

$$
\begin{array}{c}
\begin{array}{c|cccc}
 & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 & \mathcal{I}_4 \\
\hline
\mathcal{U}_1 & 1 & 0 & 1 & 0 \\
\mathcal{U}_2 & 1 & 0 & 0 & 0 \\
\mathcal{U}_3 & 0 & 0 & 0 & 1 \\
\mathcal{U}_4 & 0 & 1 & 1 & 0
\end{array} \\
X_{[1:4,*]}
\end{array}
\times
\begin{array}{c}
\begin{array}{c|cccc}
\mathcal{I}_1 & 0 & 0.41 & 0.27 & 0 \\
\mathcal{I}_2 & 0.25 & 0 & 0.81 & 0.25 \\
\mathcal{I}_3 & 0.13 & 0.17 & 0 & 0.13 \\
\mathcal{I}_4 & 0 & 0.41 & 0.27 & 0 \\
\hline
 & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 & \mathcal{I}_4
\end{array} \\
B
\end{array}
=
\begin{array}{c}
\begin{array}{c|cccc}
 & \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 & \mathcal{I}_4 \\
\hline
\mathcal{U}_1 & 0.13 & 0.58 & 0.27 & 0.13 \\
\mathcal{U}_2 & 0 & 0.41 & 0.27 & 0 \\
\mathcal{U}_3 & 0 & 0.41 & 0.27 & 0 \\
\mathcal{U}_4 & 0.38 & 0.17 & 0.81 & 0.38
\end{array} \\
\hat{X}_{[1:4,*]}
\end{array}
$$

What about explicit feedback? </>



Obrigado :) - Faculty of Information Technology